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ON THE SEMANTICAL INTERPRETATION OF LINGUISTIC ENTITIES THAT FUNCTION STRUCTURALLY*

by

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I

THIS section of the paper describes a general method, which applies the techniques of modern symbolic logic, for giving a semantical interpretation to those entities of natural language systems that function in a structural capacity. Such entities do not perform the task of denoting the individuals, properties and relations of those individuals a language talks about; they are linguistic devices that serve to combine the denotative terms into meaningful utterances. They function in many ways, such as to express syntactic relations and to express generality. Words like "each", "all", "either-or", "any", "ago" are a few examples of terms that belong in this category. Because they are the analogues in the natural language systems of logical constants in the symbolic language systems, the author has chosen to call them "structural constants". This grammatical category was first defined by H. Reichenbach in his book *Elements of Symbolic Logic* where he pointed out the necessity of distinguishing terms that function logically from those that function denotatively.*/.

The kind of definition required for giving an interpretation of these linguistic entities differs radically from the kind of definition found

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*/ See Hans Reichenbach *Elements of Symbolic Logic:* Section VII *Analysis of Conversational Language,* MacMillan New York 1947 for a detailed discussion He called these entities "logical terms", and analysed many of them. This paper is a continuation of the fundamental ideas laid down in his book.

in a dictionary that gives the meaning of a word or phrase in terms of synonyms. A structural constant is an entity whose meaning we know only when we know how to use it properly; therefore it can be defined only by a set of rules determining its behaviour. Since part of its behaviour is its interconnectedness with other structural constants, these sets of rules have to be compatible with one another, therefore the structural constant has to be defined by means of an interlocking system #.

The role that the logical constant plays in a logical system corresponds closely to the role that a linguistic structural constant plays in a natural language system. Since the logical constant is defined by a compatible complex of rules, namely the rules of the system to which it belongs, it appears to be a useful procedure to make use of the already constructed logical systems to define the structural constant. One can achieve such a definition by co-ordinating-by-definition the linguistic entity to a logical constant that corresponds to it; the structural constant is then interpreted semantically, i.e., it is defined as having the meaning (the properties) of the already defined logical constant. However, since the meaning of the linguistic entity is determined by the structure of the logical constant with respect to its logical system, the co-ordination has to be justified empirically. A natural language system is a physical phenomenon; the properties of the structural constant can be discovered only by observation. The statement that a given structural constant has all of the properties of its co-ordinated logical constant is an empirical one, therefore if the structural constant does not in fact have these properties, the statement is false and the semantical interpretation unjustifiable. However, if it can be demonstrated empirically that the interpretation holds, then a useful definition has been given.

The decision to make a given coordinative-definition cannot be arrived at mechanically. One must know the rules of the logical system well and have a good intuitive knowledge of the natural language system. Then, too, the first decision involves many entailed decisions about the co-ordinative-definitions of other structural constants. A clever initial co-ordinative-definition can solve a complex of problems and provide insight for the later ones.

\# The fact that the meaning of the structural constant can be given only by an interlocking combination of rules involving other structural constants, shows, on the basis of semantical considerations, why an immediate-constituent grammar is inadequate for generating all of the sentences of a given language system. Such a grammar is based upon the assumption that the structural units composing a sentence are independent. The definitions of the structural constants demonstrates that such independence does not always exist; a transformational grammar (cf. N. Chomsky, *Syntactic Structures*, The Hague, Mouton and Company 1957) is the only kind proposed so far that would have sufficient power to handle these structures.

However, as anyone who has attempted to translate the complicated structure of a natural language system into a given logical system can testify, the natural language is too complicated, too rich in structural devices, for any one logical system to serve as a model. The natural language system is not just one logical system; it is complex of many; the truth-functional predicate calculus, the probability calculus, modal logics, the meta-languages, and some that are not yet constructed. Structural constants can be co-ordinated to the logical constants in all of the above systems. Furthermore, the existing systems are inadequate to handle many if not most of the structures of a natural language. Motivated constructions will have to be added, in a consistent manner, to the already existing systems and new systems introduced in order to account for many of the grammatical devices.* The analyses that have been made are but a small beginning.

This method of co-ordinating linguistic entities to logical entities is a powerful aid for semantical analysis of natural language systems. It supplies us, not only with the kind of rigorous definition we need, but it provides us with a discovery technique for locating those entities that function as logical constants, whose grammatical function may have hitherto been misunderstood. Because, through the logical symbolism, the investigator can formulate schematically the sentences of a natural language system, expressing symbolically the structural constants, he is able to test the co-ordinative-definitions he sets up between the natural language system and the logical system by applying the derivation rules of that system to specific schematic formulations; if the multiple co-ordinative-definitions are correct, a derived sentence from an initial sentence containing a particular combination of structural constants should be semantically equivalent to the original statement. He can test the co-ordinative-definition in other ways too: for example, he can systematically vary the structural environment of a given structural constant for the purpose of recognizing its different meanings, since, as is well-known, natural language systems frequently use the same symbol to represent different entities; he can also vary the denotative environment to observe the effect of different kinds of denotative terms on the meaning of the structural constant.

The value of achieving correct co-ordinations between entities of a natural language system and those of a logical system is evident for purposes of mechanical translation, for if structural constants, each belonging to different language systems, can justifiably be co-ordinated to the same logical constant, they can be co-ordinated to each other since they have a common definition. Since it is the rules for combining these structural constants that must be explicitly formulated for both the input

* The author is currently working on a system for handling the semantic features of the time-structure of English.

and the target language in order to equate semantically whole sentences to whole sentences, so that a sentence-by-sentence translation can be achieved, this method provides us with the means of setting up these "equations".

II

Let us now turn to a practical application of the method described in Section I.

An obvious similarity exists between the connectives "either-or" in English and "entweder-oder" in German. Let us co-ordinate both terms to the logical operation of inclusive disjunction in the truth-functional propositional calculus, "either" and "entweder" functioning as opening parenthesis of that binary construction. "Neither-nor" and "not-(either-or)", and "weder-noch" and "nicht-(entweder-oder)" will be co-ordinated to the combinations of the two logical constants, the negation operating upon the disjunction. Symbolically,*

(1) $a \vee b$ either a or b entweder a oder b

(2) $\overline{a \vee b}$ $\begin{cases} \text{not-(either a or b)} \\ \text{neither a nor b} \end{cases}$ $\begin{cases} \text{nicht-(entweder a oder b)} \\ \text{weder a noch b} \end{cases}$

These co-ordinative definitions do not exhaust the number of meanings these words have. In both English and German the same term can frequently be co-ordinated to several unlike logical entities; it has, then, as many meanings as co-ordinations. For example, in both English and German no differentiation is made between the exclusive and inclusive disjunction - a fact which leads to a basic ambiguity in the meaning of these words. In Polish, different words represent the two kinds of disjunction, although the ordinary speaker of Polish does not always make use of the distinction.

The word "either", similarly "entweder" in German, may be deleted if its function of indicating the first term of a disjunctive expression is not required by the structural context. On the other hand, in English, the word "either" often appears without its complementary part "or", as in compound sentences of the type "a.b-either". Yet, because of its co-ordinated definition, its occurrence in this structural context makes it possible to reconstruct unambiguously the deleted parts. In a sentence like,

(3) He didn't eat breakfast and he didn't eat lunch either. #

the fully expressed symbolic formulation becomes

(4) $\overline{a}.\ \overline{(b \vee a)}$

because "either" must, by definition, indicate the first term of a dis-

* The elementary sentences represented by the variables are always indicative sentences.

\# The structural constants will always be underlined.

junction, and "not" when it precedes "either-or" must operate upon the disjunction. The proper reconstruction of the second term in the disjunction is known from the first term in the conjunction, the major connective. The negation, operating upon the first term of the conjunction, is part of the structural context, so it must remain intact, hence a is known to be the deleted right-hand term in the disjunction, not $\bar{a}$. (4) is read, "He didn't eat breakfast and (neither did he eat lunch nor did he eat breakfast)." A familiar logical theorem tells us that (4) is semantically equivalent to

(5) $\bar{a}.\ \bar{b}.\ \bar{a}.$

which is equivalent to

(6) $\bar{a}.\bar{b}$

where the redundant "$\bar{a}$" is dropped. Since, "$\bar{a}.\ \bar{b}.$" is logically, although not syntactically, equivalent to "$\overline{a \vee b}$", the informational content of (6) is equivalent to (3) with "either" deleted. Psychologically they are, of course, not equivalent, for a certain emphasis has been achieved by the introduction of an only partially expressed but fully reconstructible redundancy.

This pragmatic function of the word "either" to achieve emphasis can be recognized because of the necessary occurrence of a negative in the first clause and a negative in the second. A language, however, need not employ an explicit and independent symbol corresponding to the operation of negation, but may use implicit means to express negation. Those languages having an independent symbol possess greater structural flexibility. The negation in the first expression, $\bar{a}$, can be expressed implicitly as part of the meaning of a denotative term, such as "doubt" for "not believe", or expressed as a negative prefix. In the first clause of this structural type, words that belong to contrary pairs, like "seldom-often" can be used instead of an explicit negation, because its contrary can be used to replace the original negative word in the reconstructed clause operated upon by the negation. Thus in the sentence

(7) He seldom ate breakfast and he didn't often eat lunch either

although the first clause has an implicit negation, "either" can be used in this pragmatic way because, in the reconstruction, "He seldom ate breakfast and he did not (either often eat lunch or often eat breakfast)" there exists a contrary "often" with which to replace "seldom". The scope of the negation in a word like "seldom" is the word itself. On the other hand, the negation in the second clause of the conjunction must be expressed independently because the scope of that negation is the disjunction itself. The scope of the negation is clearly given in a semantically equivalent version of (7) when the left half of "neither-nor" is used for emphasis.

(8) He seldom ate breakfast and neither did he often eat lunch (nor

often eat breakfast).

The rules of the German language do not permit the words "entweder" or "weder" to be used in this way. The word "auch", which corresponds to "too" and "also", is used for emphasis. These words are structurally related to the conjunction "and" and "und" respectively. An analysis, which has been made of these terms, will not be covered in this paper. Suffice it to say that "auch" is used in German for sentences containing both negative and positive clauses.

On the other hand, German does permit the right-hand part of "weder-noch" to be used in structural combination with a negation and the connective "und". From the sentence "Er hat <u>nicht</u> gefrühstückt, <u>noch</u> hat er Mittag gegessen" an unambiguous reconstruction leads to the same symbolic formulation as (4) although it is the first half of the negated disjunction that has been deleted,

(9) $\overline{a}.\ \overline{(a \vee b)}$

The definition of "noch" tells us that the disjunction itself is negated. Since "$\overline{a}.\ \overline{(a \vee b)}$" is semantically equivalent to "$\overline{a}.\ \overline{b}$", "noch" serves a similar pragmatic purpose of emphasis.

The German language, too, uses contrary pairs, to express negatives like "selten-oft" and negative prefixes. In constructions like (9) the first clause must be negative or contain a word replaceable by the negative opposite, as in,

(10) Er ass <u>selten</u> frühstück <u>noch</u> ass er <u>oft</u> zu Abend.

The English language has the corresponding construction, where "nor" fulfills the same function as "noch".

Let us now turn to an analysis of the structural context of the word "either" in a context superficially similar to the one discussed above where "either" expresses the fact that the truth assertion of the combination "a.b" is unexpected. A more precise formulation, replacing the psychological term "unexpected" by a well-defined probability concept, is: the word "either" in the structural context of "($a.\overline{b}$ either)" expresses the fact that "$a.\overline{b}$" has a low probability value. There are several structural constants in English that must be defined within the system of probability logic; for example, "but", "although", "(and) yet", "and still", are conjunctive connectives that express information about the probability of the combination "a.b" occurring. A language that develops structural constants that express the sign-user's attitude toward the probability value of statements, is making use of rules that differ greatly from the rules of a two-valued logical system. The fact that native speakers of a language know how to use the terms correctly is proof that they are aware, if unconsciously, of several fundamental probability laws, just as they are aware of fundamental laws of two-valued logic. For example, in a two-valued logical system, if "a" implies "b", then "a" cannot imply "$\bar{b}$"; in a probability logic, if "a" implies "b" with a certain probability $p (p < 1)$, then "a" implies "$\bar{b}$" with a probability q, providing the condition $p + q = 1$ is satisfied. The assumption of this last stated condition is basic to the correct usage of connectives expressing unexpectedness, or low probability. Of course, those structural constants that express probability can not be defined in terms of quantitative numerical probabilities, but they can be defined in terms of qualitative estimates. Thus the law of probability stated above can be made qualitative as follows: if "a" implies "b" with a high probability, then "a" implies "$\bar{b}$" with a low probability. It is this law that is needed to define terms expressing probability connectives.

One structural clue that helps us recognise this meaning of "either" is the absence of a negative in the first clause of the compound sentence. Let us take the sentence

(11) He bought a house and it didn't cost much either.

As in example (3) when the word "either" is deleted, just the two simple facts are asserted. From (11), however, the formulation making use of the probability implication, can be made

(12) $a.\bar{b}.\ (a \ni \bar{b})_{p = low}$

which reads, "He bought a house and it did not cost much and the probability, given that he bought a house, is low that it did not cost much". The reconstruction of the context in which a sentence like this occurs is as follows. The speaker assumes and/or has knowledge that the probability of its costing a lot to buy a house is high, and he assumes that the listener also knows that fact. He forestalls an unstated expected conclusion on the part of the listener that the house cost a lot, by replacing the unstated conclusion by an assertion that it did not cost a lot. Because the speaker knows that b is expected, he knows that $\bar{b}$ is unexpected. The words "not-either" appearing in the right-hand term of a conjunction, function to cancel an assertion of an expected positive statement, and replace it by an assertion of its unexpected negative. The presence of an explicit negation is obligatory because in this context "either" can function to replace a positive statement by a negative statement, but cannot replace a negative statement by its positive.*

* This function of the word "either" in combination with "not" to cancel a previous assertion of a positive statement and replace it by the assertion of its negative, is a use of this word that the author has never seen analyzed, perhaps because it is a word used almost entirely in conversation. Another illustration of this usage, but without the accompanying probability connecttlon is seen from the sentence "He didn't either do it"; here one can make an unambiguous reconstruction of the context that preceded that statement, namely that the sentence "He did do it" had been asserted as true. An assertion,

(continued on page 551)

The German language system does not use the word "entweder" in this way. Low probability is expressed by the phrase "(noch) nicht einmal". If "a" is "Er kaufte ein Haus" and "b" is "er hat viel dafür bezahlt, nicht" the explicit negation operating on "b", one can write

(13) Er kaufte ein Haus <u>und</u> er hat (<u>noch</u>) <u>nicht</u> <u>einmal</u> viel dafür bezahlt.

"Noch" may be deleted without a change in semantical meaning. The deletion of "einmal" however affects the meaning of the German statement in much the same way as the deletion of "either" affects its English counterpart. In translation, the German statement is usually rendered as "He bought a house and he didn't even pay much for it". The word combination "not-even" conveys also the meaning of an unexpected combination and can replace "not-either" if proper precautions as to word order are taken.

In the foregoing analyses, the structural constants discussed were co-ordinated to propositional operations. In English, the word "either" * is also used as a determiner, as in the sentence "Either road leads to London". Determiners like "any" "all" "every" "each" are defined by co-ordination to logical constants occurring in the predicate calculus where the inner structure of an elementary proposition is symbolized.

The determiner "either" is a special case of the determiner "any", "any" being defined as a free variable, because "either" has the properties of a free variable with the restriction that it ranges over only two values.

In order to clarify the following discussion, the following co-ordinative-definitions are needed:

(1) "all" is co-ordinated to the universal-quantifier "(x)" when the scope of "(x)" does not include the scope of an existential-quantifier, "(Ex)", in prenex position. "Both" is a special case of "all", and co-ordinated to a restricted universal-quantifier ranging over two variables.

(2) "every" is co-ordinated to the universal-quantifier when its scope is not included within the scope of an existential-quantifier. "Each of two"*/ is a special case of "every",

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being an act on the part or the sign-user, cannot be denied; the additional act of first cancelling the first assertion and then replacing it by the assertion of the negative has to be performed. The previous assertion has to be removed because of the rule that if "a" can be asserted and "b" can be asserted, then "a.b" can be asserted. It is obvious that "$a.\bar{a}$" cannot be asserted.

* It retains, however, its basic connection to the operation of disjunction because the definition of a free variable involves the use of a disjunction in the meta-language.

*/ "Each" is usually used interchangeably with "every" except in the case where there are only two elements.

being a restricted universal-quantifier ranging over two variable.

Normally, specific symbols standing for such restricted quantifiers do not appear in the logical calculi because such quantifiers can be expressed in terms of other primitive symbols. However, specific symbols representing logical constants have to be introduced into a logical system that is intended to be co-ordinated to the natural language systems when specific structural constants exist in that language, and rigorous rules for their occurrences, logically consistent with the already existing rules, have to be established so that precise definitions of these linguistic entities can be given. #.

Free variables, whether they are used in mathematical systems or natural language systems, have rather strange properties, inherent in the nature of a free variable, in that they can be replaced by variables bound by either the universal or existential quantifier under certain special rather complicated conditions, thus "any" can under certain circumstances be replaced by "all", or "every", and under other circumstances by "some" or "at least one" - these circumstances depending upon the structural relationship of the free variable to other structural constants, like "not", "if then" etc. Thus one can give a basic definition of such words as "any" and "either" and then demonstrate where it is permissible to replace* say, the word "either" by "both" or "one of the two", leaving the rest of the structure Intact.

These replacements are legitimate when the resulting sentence is semantically equivalent to the original sentence. It must be emphasized that the basic co-ordinative-definitions of the words that express free variables never change. It is the rules of replacement that differ. To be sure, the semantical meaning of a sentence containing the word "either" in one structural environment differs from the meaning of a sentence where "either" appears in a different structural context. When one says "I am using "either" in the meaning of "both" ", what is meant is, "The sentence containing "either" is semantically equisignificant to a similar sentence containing "both" ". This distinction may sound very pedantic, but it is

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The technical logical definitions that have been worked out will not be included in this paper.

* The word "replace" is used in this paper with a special logical meaning, and differs from the word "substitute". One can legitimately ***replace*** a structural part of a formula only by its logical equivalent. One can ***substitute*** one variable for another in a formula, if one makes the substitution in all those places where the original variable occurs. A substitution does not alter the syntactic structure, but alters the semantics; a replacement alters the syntactical structure without altering the semantical meaning.

necessary for a clear understanding of the nature of free variables because they can be bound by different quantifiers.

The German language does not possess a special symbol to express a free variable restricted to two values; in translation, therefore, particularly from English to German, one must know the rules of replacement.

Let us now turn to a demonstration that "either" has the properties of a restricted free variable.

The well-known law of generalization states that if a statement containing free variable is asserted as true, a universal-quantifier binding the free variables can be placed before the statement, the scope of the quantifier to extend over the entire statement. The semantical justification for this law is: Since a free variable represents an arbitrary selection of an element out of a set, if one can assert as true that a property holds for an arbitrary selection of a member of a set, then he must know in advance that it holds for all members of that set.

Taking the statement, "Either road leads to London", we formulate it symbolically, letting "x" be the free variable, expressing the structural constants by logical constants, and writing out the original predicate and individual constants.

(14) $\text{road}^{2*}(x) \supset \text{leads to } (x, \text{London})$

Placing the universal-quantifier in front of the formula and indicating the scope by brackets, we obtain the semantically equivalent formulation,

(15) $(x)\ [\text{Road}^{2}\ (x) \supset \text{leads to } (x, \text{London})]$

Since a generalized version of (14) serves as the logical definition of the restricted operator "both", the word "either" in (11) can be replaced by "both", accompanied of course by the syntactic change to the plural in the denotative term "road".

Let us turn to a more complicated structure where the word "either" occurs in the implicate of a conditional, "When either boy enters the library, he starts to study":

(16) $\text{boy}^{2}\ (x) \supset [\text{enters } (x, \text{library}) \supset \text{starts to study } (x)]$

Applying the law of generalisation, we obtain

(17) $(x)\ [\text{boy}^{2}\ (x).\ \text{enters } (x, \text{the library}) \supset \text{starts to study } (x)]$

By inspection of (17), we find, because of the coupling conditions attached to the use of a free variable, namely that if one once selects a free variable to represent an unknown individual, the use of the same free variable in other parts of the formula indicates that the referent is

* The superscript 2 indicates the restriction to a pair of roads that is a subset of the class of roads.

the same individual, that all of the free variables have become bound to the universal-quantifier. Thus, the property of studying once the library has been entered holds for both boys. In this structure, however, it is not permissible to replace "either" by "both", since such a replacement introduces an ambiguity, namely the sentence could mean that it is only when both enter do they begin studying. The word "either" in (16) clearly expresses the notion of the independence of the two events; i.e., when boy x_1 enters, x_1 studies and when boy x_2 enters, x_2 studies. The use of "either" is actually the best way of expressing unambiguously the meaning of this sentence as the only possible replacement is "each of the two", which is more awkward.

The fact that a variable bound to the quantifier occurs in the implicate as well as in the implicans is an important structural feature. To realize its significance, let us analyze a sentence that differs only in this one respect, an individual constant, a proper name, is substituted for the bound variable "he". Taking the sentence "When either boy enters the library, Mary starts to study". We formulate it,

(18) $\text{boy}^2\ (x) \supset [\text{enters}\ (x, \text{library}) \supset \text{starts to study}\ (\text{Mary})]$

It is obvious, from the meaning of the sentence, that "either" appears to have undergone a change of meaning. Clearly, the word "both" cannot replace "either", because here "either" has the meaning of "one or the other of the two". Let us now demonstrate that it is the change in structural context that has altered the replacement rules, not a change in the definition of "either" as a restricted free variable. Applying the law of generalisation and "$a \supset (b \supset c) \equiv (a.b \supset c)$" to (18), we write,

(19) $(x)\ [\text{boy}^2\ (x).\ \text{enters}\ (x,\text{library}) \supset \text{starts to study}\ (\text{Mary})]$

Since the scope of the universal-quantifier is the whole formula, (19) is semantically equivalent to

(20) $[(Ex)\ \text{boy}^2\ (x),\ \text{enters}\ (x,\text{library}]\ \text{starts to study}\ (\text{Mary})$

where the scope of the existential-quantifier extends only over the implicans. (20) reads "if at least one of the two boys enters the library, Mary starts to study". Because of the restriction on the existential-quantifier to two, "either" has the meaning of "either one or the other". (19) and (20) are semantically equivalent because the implicate is constant with respect to the implicans; in our example, whichever event occurs, x_1 entering the library or x_2 entering the library, Mary starts to study. Symbolically stated, if "$a \supset c$" and "$b \supset c$", then "$a \vee b \supset c$". The replacement rule is formulated symbolically,

(21) $(x)\ [f^2\ (x) \supset a] \equiv [(Ex)\ f^2\ (x)] \supset a$

where "a" is represented as a propositional variable, constant with respect to the implicans.

An existential-quantifier restricted to two variables can easily be defined in the predicate calculus, but no special symbol corresponding to this concept occurs in English, hence "either" must be replaced by a phrase. A language that has developed a special word can easily be imagined.

By examining a still more complicated structural context, it can further be demonstrated that "either" does in fact have the fundamental properties of the restricted free variable. Taking the sentence "There is a fireplace at either end of the hall", formulating it on free variables, we write

(22) end^2 (x, the hall) ⊃ (Ey) fireplace (y). located at (y.x)

Applying the law of generalisation, and putting the existential-quantifier in prenex form in accordance with the theorem,
"(x) [f(x) ⊃ (Ey) g(y).h(y,x)]" is equivalent to
"(x)(Ey) [f(x) ⊃ g(y).h(y,x)]", we obtain,

(23) (x) (Ey) [end^2 (x, the hall) ⊃ fireplace (y). located at (y,x)]

Inspection of (23) shows that the universal-quantifier binds the x's occurring in both the implicate and the implicans, hence the implicate cannot be regarded as a constant with respect to the implicans and consequently the theorem (21) does not apply; i.e., it cannot be derived that at least one of the two sides has a fireplace. Nor does the sentence convey that information. Yet, the word "both" cannot replace "either" because such a replacement introduces an ambiguity. In (23) both kinds of quantifiers appear, the universal quantifier preceding the existential quantifier, thus the scope of "(x)" includes that of "(Ey)". This particular ordering of quantifiers means that there exists a y for every x. Since the universal-quantifier is restricted to two variables in our example, this ordering of the quantifiers means that there exists a fireplace for each of the two sides. The word "each", as does the word "every" when no restriction as to number is made, performs this distributive function, that of co-ordinating an element from one set to an element from another set by means of a mapping relation (in our example, the binary predicate relation "located on") in addition to its function of determining the totality of elements to be so co-ordinated. When the quantifiers are reversed as in

(24) [(Ey) fireplace (y)].(x) [end^2 (x, the hall) ⊃ located (x,y)]

the formulation reads, "There is a fireplace on both ends of the hall", which means, strictly speaking, that there is one fireplace for both. This confusion of "both" for "each of two" when the distributive function of "each of two" is required is very common. Common sense tells us in this case that two ends separated by space cannot share one fireplace, but in many cases a genuine ambiguity is introduced because the correct rules are not followed. The use of the free variable "either" avoids this ambiguity.

In German, the word "jeder" is interpreted as having the same structural properties as "every", and "alle" the same as "all". There is no corresponding special term for "each" except the phrase "jeder der beiden".

Further proof that the word "either" has the properties of a free variable lies in its behaviour when appearing in a question. "Does either road lead to London?" has the meaning of asking whether there is one of the two that has the required property. It is only when a sentence containing "either" can be asserted as true that it can be replaced by "both" or "each".

Free variables occurring in statements containing a negation have the following property: If the statement is asserted that an arbitrary member of a set F does not have a property g, then the statement "there does not exist a member of F that has the property of g" can be asserted as semantically equivalent. This theorem is a consequence of the law of generalisation. The sentence, "John does not want either book" illustrates that "either" behaves structurally according to the above rule for free variables.

(25) $\text{book}^2\ (x) \supset \overline{\text{wants (John, } x)}$

Applying the law of generalisation,

(26) $(x)\ [\text{book}^2\ (x) \supset \overline{\text{wants (John, } x)}]$

Applying "$(a \supset b) = \overline{a.b}$" and "$(x)\ \overline{f(x)}$ a $\overline{(Ex)f(x)}$", we can write,

(27) $\overline{(Ex)\ \text{book}^2\ (x).\ \text{wants (John, } x)}$

which states the semantically equivalent, "There does not exist one of the two books that John wants".

The sentence "John wants neither book" corresponds structurally to (27) because the special symbol "neither" is a contraction of two logical operators, a negation whose scope includes a restricted-existential-quantifiers ranging over two variables. "No" as in "No boy likes that" is a similar contraction except that the existential-quantifier is not restricted in number. "Neither" therefore, can never, by definition, function as a negated free variable. In German, the word "weder" does not function as a special symbol representing a contraction of these two operators. The phrase "weder ein noch der andere" must replace "neither".